A MONTE CARLO SIMULATION MODEL OF INTER-VEHICLE COMMUNICATION

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ABSTRACT

De-centralized traffic information systems based on inter-vehicle communication have drawn increasing attention in recent years. In this paper, based on the assumption that inter-vehicle communication is instantaneous relative to vehicle movement, we study multihop connectivity between two equipped vehicles subject to arbitrary distribution patterns of vehicles, market penetration rates, and transmission ranges. After discussing the modeling conceptual framework and definitions, we present a Monte Carlo simulation model of multihop connectivity of instantaneous inter-vehicle communication systems. With three different, well-chosen random number generators, we demonstrate that the Monte Carlo simulation model yields results both consistent with those in literature that consider vehicle mobility and which cross-validate analytical models developed previously. The features of the simulation model facilitate determination of the connectivity of large-scale inter-vehicle communication systems.

1. INTRODUCTION

In recent years, *inter-vehicle communication* (IVC) based on such wireless communication technologies as Wi-Fi has become an intriguing option in developing intelligent transportation systems. With a subpopulation of vehicles equipped with wireless communication and information processing units, real-time critical traffic information can be propagated both to relevant travelers to help make proper choices over routes and departure times and also to traffic management centers to help operate the traffic system more efficiently. With these transportation-related applications in mind, researchers at the California Institute of Telecommunications and Information Technology and the Institute of Transportation Studies of the University of California, Irvine, have been engaged in comprehensive research efforts aimed at the development of an autonomous, self-organizing, transportation management, information, and control system (Autonet). Autonet-like system could be the basis of next generation traffic information systems that are decentralized in nature and more resilient to such disasters as earthquakes.

With equipped vehicles as communication nodes, an IVC system can be considered as a special mobile ad hoc network *(1)*, whose topology is highly dynamic *(2,3)*, since vehicles can enter or leave a road section, the relative position of two vehicles traveling in opposite directions can change rapidly (as fast as 240 km/h), and even the relative positions of vehicles traveling in the same direction can fluctuate due to passing, braking and accelerating maneuvers in a traffic stream. In this sense, it may be difficult to maintain a stable communication route between two nodes. It has been argued that broadcasting would be sufficient for distributing useful traffic information for most transportation related applications *(4)*, and, therefore, dynamic communication topology would not be a serious issue. In fact, the movement of vehicles could be quite beneficial for relaying traffic information, particularly when the market penetration rate of equipped vehicles is low at early stages of deployment, since a piece of information can be propagated to upstream vehicles by a vehicle traveling in the opposite direction or to a whole road network through merging and diverging vehicles. A disadvantage of such information propagation is the associated delay, limited by the relative speeds of the respective traffic streams, which prevents its applicability for such uses as immediate incident warning *(5)*.

When the market penetration rate is sufficiently high, a more efficient approach to relaying information in an IVC system is through multihop broadcasting. In this manner, a piece of information can be transmitted to vehicles outside of communication range in a relatively short time. It is reasonable to assume such multihop broadcasting to be *instantaneous* with respect to vehicle movement. For example, a packet of 73 bytes can be transmitted once in 110 ms with a communication bandwidth of 3.6 kb/s *(6)*; i.e., it takes only 1.1 s for information to reach as far as 5 km with a communication range of 500 m. In that same 1.1 seconds, the maximum change in the relative positions of vehicles in the same or opposition directions is about 73 m (with a relative speed of 240 km/h). The assumption of instantaneity is increasingly valid as traffic becomes congested, where vehicle speed is significantly lower. Under such conditions, *instantaneous IVC* (IIVC) can effectively extend the range of line of sight of vehicles as real-time changeable message signs *(7)*. For example, IIVC could be applied in incident scenarios, where a short, high-priority message related to the occurrence of incident can be propagated virtually instantaneously to following vehicles so that they have sufficient time to correspondingly decelerate, change lanes, or switch routes.

The connectivity between two nodes is an important characteristic of information propagation in a wireless communication system. Although there have been extensive analytical studies of the connectivity of stationary wireless networks *(e.g. 8)*, these studies have assumed that communication nodes are uniformly randomly distributed on a line or a plane. Recently, Wu et al. *(9)* modeled information propagation through IVC and considered the effect of vehicle movement on information propagation, assuming Poisson arrival of vehicles and random and independent vehicle mobility. However, vehicles in a road network usually do not follow spatial Poisson distribution, since their positions are inter-dependent and evolve in certain patterns caused by restrictions of road geometry, traffic signals, and car-following rules *(10)*. For example, in the vicinity of a lane-drop location or merging junction, traffic is usually more congested on the upstream part; gaps between platoons of vehicles on surface streets constantly change due to traffic signal controls; shock waves *(11)* formed by the closure of a lane due to an incident typically result greater vehicle density downstream of the shock wave interface. Since these situations are precisely those for which real-time traffic information can be beneficial, it is important to analyze such non-Poisson distribution patterns of vehicles on a road both when considering the performance of an IVC system or in determining appropriate technologies to implement an IVC system. In other studies of IVC systems, traffic simulators were used to predict the locations of vehicles, and the connectivity was estimated through repeated simulations with randomly-selected equipped vehicles *(12,13,14)*. However, due to the complexity in traffic dynamics, this approach is usually very time consuming and not appropriate for analyzing large road networks.

In this study, we allow for arbitrary locations of vehicles, under the assumption of instantaneous information propagation. That is, we consider an IIVC system as a stationary wireless network with deterministic node positions that, at any instant, are defined by the particular traffic flow conditions. We first carefully analyze the conceptual framework underlying IIVC and show that it is natural to use Monte Carlo simulations *(15)*, where the probability for one vehicle to be equipped is determined by market penetration rate, to study the properties of an IIVC system. The new model has simpler concepts than the analytical models of IIVC developed in *(16,17)* and can be used to verify the analytical results of this and other such models. The features of the new model enable it to be an efficient alternative approach to analytical models.

The rest of the paper is organized as follows. In Section 2, we discuss the conceptual framework of and definitions associated with IIVC. In Section 3, we introduce a Monte Carlo simulation model and three well-chosen random number generators, vital to Monte Carlo simulations. In Section 4, we study the effect of transmission range and traffic density on multihop connectivity and compare simulation results with theoretical results. In Section 5, we offer conclusions.

2. CONCEPTUAL FRAMEWORK AND DEFINITIONS

In a road network, given appropriate initial and boundary conditions, vehicle positions at any time can be predicted by anyone of a number of available traffic models, such as kinematic wave models *(18)*. At any time instant, we can then take a “snap-shot" of a traffic stream and analyze information propagation through IIVC. That is, in an IIVC system, positions of vehicles are determined by traffic models, and the *distribution pattern of vehicles* can be arbitrary (either uniform, in which the distances between two consecutive vehicles remain the same, or non-uniform). Although the positions of vehicles are deterministic in IIVC, the probability for any individual vehicle to be equipped remains the same as *market penetration rate*, μ, i.e., the percentage of equipped vehicles among all. Assuming whether or not any two particular vehicles are equipped is independent, the distribution of equipped vehicles in a traffic stream can be described by *Bernoulli trials (19, Chapter VI)*. Then for a traffic stream of K vehicles, there can be 2^K different Bernoulli trials. A *realization* of Bernoulli trials corresponding to a traffic stream can be represented by a sequence of 0's and 1's, where 0's stand for non-equipped vehicles and 1's for equipped vehicles, and all possible realizations can be represented by integers from 0 to $2^K - 1$. The probability of the existence of a given Bernoulli trial can be computed from market penetration rate.

To determine how far information can propagate in a realization of Bernoulli trials, we can construct a so-called *"most forwarded within range" (MFR) (20)* communication chain of *nodes* as follows. First, we denote the information source by node 0. Then, within the *transmission range*, R, i.e., the maximum transmission distance at one hop, of node h ($h \geq 0$), node $h+1$ is the equipped vehicle farthest in the direction of information propagation. We can see that for each realization of Bernoulli trials, there exists one and only one MFR communication chain. Although it is possible to arrive at node $h+1$ by more than one chain, an MFR communication chain is defined as the most efficient communication scheme in the sense that it takes the smallest number of hops to transmit a message from the source to a receiver. That is, in our study, we do not consider how information is actually propagated, and it is not necessary for individual vehicles to have MFR knowledge. For example, each vehicle can just broadcast their information, and the farthest reach of the information through multihop IVC is determined by the MFR communication chain. Hereafter, an MFR communication chain is simply called a *communication chain*.

In an IIVC system, two equipped vehicles are connected if and only if there exists a communication chain between them. That is, the connectivity between two equipped vehicles is merely determined by the probability of the existence of a communication chain between them. Obviously, the probability of the existence of a communication chain is determined by relative locations of vehicles, transmission range, and the market penetration rate.

2.1. Definitions

In our study, vehicles are labeled in order according to their distances from the information source, vehicle 0. That is, the distance from the information source is not smaller for vehicle $k+1$ than vehicle k. Note that, two vehicles shoulder-to-shoulder on a multi-lane road are assigned different orders, and different ways of assigning orders to these vehicles should not affect the connectivity property. Further, we denote the farthest vehicle within the transmission range of vehicle k in the downstream direction of information direction by k^*, and that in the upstream direction by k_*.

We denote by $(k;h)$ the event that vehicle k is node h in a communication chain. If node h is the *end node*, from which information cannot be further relayed, we denote the event by $(k;\bar{h})$. Then, the probability for vehicle k to be node h of all possible communication chains is denoted by $P(k;h)$, and the probability for vehicle k to be end node h by $P(k;\bar{h})$. Here $P(k;h)$ is called *node probability*, and $P(k;\bar{h})$ *end node probability*. Since both $P(k;\bar{h})$ and $P(k;h)$ are hop-related quantities, we can define many performance measurements in terms of relay structure *(16,17)*. In this study, we only consider *connectivity* between information source and an equipped vehicle. If we define $P(\bar{k})$ as the probability for a communication chain to stop at vehicle k, regardless of the number of hops, then

$$P(\bar{k}) = \sum_h P(k;\bar{h}), \tag{1a}$$

and the probability for information to travel to and beyond vehicle k a traffic stream of K vehicles (excluding the information source) is given by

$$s(k) = \sum_{i=k}^{K} P(\bar{i}). \tag{1b}$$

Further, if vehicle k is equipped, then it is connected to the information source as long as a communication chain does not end before k_*, the farthest upstream vehicle of k. Therefore, the connectivity between the information source and equipped vehicle k equals the probability for information to travel beyond vehicle k_*,

$$\tau(k) = s(k_*). \tag{1c}$$

We can also define the connectivity between two equipped road-side stations as the probability of IIVC communication chain connecting them.

2.2. Analytical models

In the analytical models of connectivity of IIVC in *(16,17)*, a new concept of *(transmission) cells* was introduced. With the same transmission range R for all wireless units, a traffic stream can be split into cells as follows: All vehicles within the transmission range of, but not including, the information source belong to cell 1; all vehicles within the transmission range of, but not including, any vehicle in cell c belong to cell $c+1$. Vehicle k in cell c can also be denoted by (c,k), and the information source by $(0,0)$. Then the node probability, i.e., the probability for vehicle (c,k) to be node h, and the end node probability, i.e., the probability for vehicle (c,k) to be an end node h, can also be denoted by $P(c,k;h)$ and $P(c,k;\bar{h})$, respectively. That is, $P(c,k;h) \equiv P(k;h)$, and $P(c,k;\bar{h}) \equiv P(k;\bar{h})$.

Based on the analysis of the regulatory properties of communication chains spanning over cells, regressive models of node or end node probabilities were given as follows *(16,17)*. First, quantities in cell 1 can be computed by:

$$\begin{aligned} P(1,k;1) &= P_1(1,0,k;1) = \mu \nu^{n_1-k}, \quad \text{for } k = 1,\cdots,n_1; \\ P_2(1,l,i;1) &= 0, \quad \text{for } 1 \le l < i \le n_1; \end{aligned} \tag{2a}$$

where μ is the market penetration rate, and $\nu = 1-\mu$. Then, given quantities in cell c, quantities in cell $c+1$ can be obtained regressively:

$$P_1(c+1,i,k;h+1) \quad = \quad \left(P(c,i;h) - \sum_{l=k_*}^{i-1} P_2(c,l,i;h) \right) \mu \nu^{i^*-k},$$

$$\text{for } i = k_*,\cdots,n_c; k = n_c+1,\cdots,n_{c+1}; h = c,\cdots,2c-1;$$

$$P_2(c+1,j,k;h+1) \quad = \quad \sum_{i=j_*}^{k_*-1} \left(P(c,i;h-1) - \sum_{l=j_*}^{i-1} P_2(c,l,i;h-1) \right) \mu^2 \nu^{i^*-j-k+j^*}, \qquad \text{(2b)}$$

$$\text{for } j = n_c+1,\cdots,k-1; k = n_c+2,\cdots,n_{c+1}; h-1 = c, \quad , \quad c- \;;$$

$$P(c+1,k;h+1) \quad = \quad \sum_{i=k_*}^{n_c} P_1(c+1,i,k;h+1) + \sum_{j=1}^{k-1} P_2(c+1,j,k;h+1),$$

$$\text{for } k = n_c+1,\cdots,n_{c+1}; h = c+1,\cdots,2c+1.$$

In each cell, end node probabilities are computed by:

$$P(c,k;\overline{h}) = P(c,k;h) - \sum_{i=k+1}^{n_c} P_2(c,k,i;h+1) - \sum_{i=n_{c+1}}^{k^*} P_1(c+1,k,i;h+1). \qquad \text{(2c)}$$

Here n_c is the farthest vehicle in cell c, and $P_1(c+1,i,k;h+1)$ and $P_2(c+1,j,k;h+1)$ are hop probabilities, or joint probabilities of two consecutive nodes.

Note that node or end node probabilities are computed from hop probabilities. The cost for computing hop probabilities is linear to the product of the number of cells, the square of the number of vehicles in each cell, and the number of hops, linear to the number of cells. Thus, the computational load of this analytical model is quadratic to the total number of vehicles.

3. MONTE CARLO SIMULATIONS AND RANDOM NUMBER GENERATORS

Whether or not a vehicle is equipped is random in an IIVC system. To evaluate node and end node probabilities with predefined vehicle locations and communication range, one natural idea is to carry out Monte Carlo simulations through repeated, random realizations of Bernoulli trials. Since in each Bernoulli trial, whether a vehicle is a node or end node of a communication chain is deterministic, its average over the number of experiments would be an estimation of the corresponding node or end node probability. Compared to analytical approaches, the Monte Carlo simulation model does not require understanding of the structure of communication chains with respect to transmission cells. Thus, we can eliminate cell-membership and ignore hop probabilities, but are still able to compute hop-specific node probabilities, $P(k;h)$ and $P(k;\overline{h})$. In this regard, the conceptual framework underlying Monte Carlo simulations is simpler than that employed in the analytical model.

3.1. A Monte Carlo simulation model

For a traffic stream with K vehicles, the flow-chart of Monte Carlo simulations of instantaneous inter-vehicle communication is shown in Figure 1, in which each step is carried out as follows.

- Inputs for the experiments include positions of K vehicles $x(k)$

($k = 1,\cdots,K$), penetration rate μ, communication range R, and the number of experiments M. Note that the position of information source is $x(0) = 0$.

- In each experiment, K uniformly distributed random variables, among which random variable X_k corresponds to vehicle k, are generated in $[0,1]$. Here the interval can also be $(0,1)$, $(0,1]$, or $[0,1)$.
- If $X_k \le \mu$, vehicle k is equipped, and we can obtain a realization of Bernoulli trials. Here the event when vehicle k is equipped is denoted by $(k;-1)$.
- In each realization of Bernoulli trials, starting from the source, information is transmitted to the farthest equipped vehicle within a communication range, i.e., in the manner of MFR. In this way, we can obtain a unique MFR communication chain for each experiment. If information travels to vehicle k with h hops ($h > 0$), or vehicle k is the hth node of a communication chain, we denote this event by $(k;h)$. Further, if node h is the end node of the MFR communication chain, we denote the event by $(k;\bar{h})$.
- The numbers of occurrences of $(k;-1)$, $(k;h)$, and $(k;\bar{h})$ are denoted by $N(k;-1)$, $N(k;h)$, and $N(k;\bar{h})$, respectively. In each experiment, these quantities are added by 1 if the corresponding events occur.

After finishing M experiments, we can compute the simulated probabilities of $P_M(k;-1)$, $P_M(k;h)$, and $P_M(k;\bar{h})$ respectively by

$$P_M(k;-1) = \frac{N(k;-1)}{M}, \tag{3a}$$

$$P_M(k;h) = \frac{N(k;h)}{M}, \tag{3b}$$

$$P_M(k;\bar{h}) = \frac{N(k;\bar{h})}{M}. \tag{3c}$$

Then according to Equations 1, we can use $P_M(k;\bar{h})$ to compute $P_M(\bar{k})$, $s_M(k)$, and $\tau_M(k)$, respectively.

Here $P_M(k;-1)$ is a Monte Carlo estimation of the market penetration rate, μ. To evaluate the accuracy of the Monte Carlo estimation of μ, we define the following three aggregate errors between $P_M(k;-1)$ and μ:

$$\|P_M(k;-1) - \mu\|_1 = \frac{\sum_{k=1}^{K} | P_M(k;-1) - \mu |}{K},$$

$$\|P_M(k;-1) - \mu\|_2 = \sqrt{\frac{\sum_{k=1}^{K} | P_M(k;-1) - \mu |^2}{K}},$$

$$\|P_M(k;-1) - \mu\|_\infty = \max_{k=1}^{K} | P_M(k;-1) - \mu |.$$

Similarly, node and end node probabilities $P_M(k;h)$ and $P_M(k;\bar{h})$ are Monte Carlo estimations of $P(k;h)$ and $P(k;\bar{h})$ obtained from the analytical model, (2), respectively. Hence, we can use the above errors to evaluate the accuracy of Monte Carlo estimations of both node probabilities and end node probabilities. Note that both $P(k;h)$ and $P(k;\bar{h})$ are two-dimensional vectors, and the summation in the equations above has to be taken with respect to both the number of vehicles and the number of hops.

Since, for the Monte Carlo simulation model, the computational time is linear to the total number of vehicles and the number of simulations, the model is an efficient alternative to the analytical model in Equations 2, whose computational load is quadratic to the number of vehicles for a very large number of vehicles. Note that approaches in *(12,13)* can both be considered Monte Carlo simulation models. In these studies, however, the interactions between vehicle movements and information propagation are considered by using traffic dynamics models, only a limited number of experiments were carried out due to high computational cost, and the convergence of the Monte Carlo simulations was not discussed.

3.2. Three random number generators

To ensure the validity of results of Monte Carlo simulations, general requirements on *random number generators* (RNGs) include long period, good distribution properties, and fast computing speed. Since every RNG has its own known or unknown limitations and could intrinsically interfere with a Monte carlo simulation model, chosen RNGs should be thoroughly analyzed and tested theoretically and empirically.
In our study, we employed three different RNGs for comparison purposes: the standard RAND() function included in C-library stdlib.h, MRG32k3a *(21)*, and MT19937 *(22)*. The first is a simple C-function and can be easily implemented, while the latter two, with carefully chosen parameters, have been theoretically and empirically shown to have long periods, good distribution properties, and fast computational properties. In the following, the three RNGs are briefly introduced.

1. RAND is a standard RNG in the C-library stdlib.h and realized by a linear congruential method *(23)*. On a 32-bit machine, the period of this RNG is about 2^{31} *(22)*.
2. The MRG32k3a *(21)* algorithm, a combined multiple recursive random number generator (MRG), combines three copies of multiple recursive random number generator. The MRG32k3a RNG returns a uniformly distributed random number in $(0,1)$, with period length of 2^{191}. All quantities in the MRG generator can always be exactly represented by 32-bit floating point numbers on a computer supporting the IEEE 754 floating-point arithmetic standard, on which a number of double precision has 53 bits for the significand *(24)*. Therefore, in the implementation of MRG32k3a, all variables are converted to exact floating-point representations (e.g., in C-language compiler), and all computations are directly done with floating-point arithmetic. This implementation is generally faster than implementation with integer arithmetic. Note that, however, even though generated random numbers are represented with 53-bit floating-point numbers, the actual output strings are just 32 bits due to rounding errors, and, to obtain more precision, one can combine two or more strings together. In *(21)*, the MRG32k3a RNG has been shown to have good structural properties through spectral test *(23)*.
3. The MT19937 RNG, Mersenne Twister 19937 *(22)*, is a type of multiple-recursive matrix method based on boolean arithmetics. MT19937 has 623-dimensional equidistribution property and an exceedingly large period of $2^{19937}-1$, where 19937 is the 24th known Mersenne number *(25)*. Due to computational difficulty of implementing

spectral test for the 623-dimensional RNG, k-distribution test was used as the major statistical test. In MT19937, favorable k-distribution property is obtained. In addition, MT19937 also passed other standard tests, such as the diehard tests *(26)*. With the aforementioned features, MT19937, consuming 624 computer word, was reported to be as fast as RAND since it is based on boolean arithmetics. Therefore, it is suggested that MT19937 could be most suitable for Monte Carlo simulations of complicated systems *(22)*. In our study, we use an implementation of MT19937 in C *(27)*.

4. MONTE CARLO SIMULATION RESULTS

In this section, we present numerical results of the Monte Carlo simulation model using the three different RNGs, whose seed numbers are in the following: 12345 for initial value x_1 in RAND, 12345 for all initial values of $(x_{1,0}, x_{1,1}, x_{1,2})$ and $(x_{2,0}, x_{2,1}, x_{2,2})$ in MRG32k3a, and four initial values 0x123, 0x234, 0x345, 0x456 in MT19937.

4.1. Multihop connectivity in uniform traffic

For a traffic stream that is 10 km in length and with a penetration rate μ=10%, we use the Monte Carlo simulation model to study multihop connectivity between an equipped vehicle at x and the information source at 0 for traffic densities $\rho = 58$ and 19 veh/km and communication ranges R=1000, 500, 200, and 100m. Here we use the MT19937 RNG with 1000 simulation runs. Note that, in Figures 1 and 2 of *(12)*, connectivity was studied for two traffic streams are on 2×2 lanes with average distances between two vehicles at 69 m and 208 m; i.e., the total densities were approximately $58 \approx 1000/69 \cdot 4$ and $19 \approx 1000/208 \cdot 4$ veh/km. Therefore, the scenarios studied here are almost identical to those in *(12)*, except that vehicle mobility was included in the latter but not in the Monte Carlo simulation model of IIVC.

In Figures 2 and 3, the connectivity between an equipped vehicle and the information source is shown for uniform traffic streams with densities of 58 and 19 veh/km, respectively. As expected, connectivity increases with transmission range and traffic density. From these figures, which also show the results of *(12)*, we can observe that the multihop connectivity computed by the Monte Carlo simulation model is consistent with that found by Hartenstein et al. *(12)*. This consistency argues for the acceptability of the assumption of instantaneous information propagation in a dynamic traffic stream. However, the effect of vehicle mobility can be ignored only when the market penetration rate is relatively large as in the scenarios studied here.

4.2. Comparison of the Monte Carlo simulation model and the analytical model

With penetration rate $\mu = 10\,\%$ and communication range $R = 1$ km, we compare Monte Carlo simulation results with those obtained from the analytical model in Equations 2. The number of experiments vary from 10^3 to 10^8, and the CPU times reflect the performance of the Monte Carlo simulation model running on a 32-bit Windows-xp desktop with 3GHz Pentium-4 CPU and 1GB Ram, using MINGW32 C/C++ compiler *(28)*.

We first study IIVC in a randomly generated traffic stream, comprising 15 communication cells and 1422 vehicles, where the number of vehicles in each communication cell is a uniform random number between 75 and 125. We show the differences in penetration rate and node probabilities between theoretical and Monte Carlo simulation models in Tables 1, 2, and 3. From these three tables, we have the following observations. First, the three RNGs yield almost the same errors in penetration rate μ and $P(k;h)$, but MRG32k3a takes almost twice the time as RAND and MT19937. Thus the RNG of MT19937 is more desirable for the Monte Carlo simulation model of IIVC. Second, the differences, e.g. $\|P_M(k;-1)-\mu\|_1$ and $\|P_M(k;h)-P(k;h)\|_1$, decrease with increasing number of samples in a rate proportional to $1/\sqrt{10}$. The fundamental theorem of Monte Carlo simulations states that valid Monte Carlo simulation results converge to the exact values in a rate inversely proportional to the square-root of the number of simulations *(29, Chapter 4)*.

Then for a uniform traffic stream, comprising 15 communication cells and 300 vehicles, we compare the Monte Carlo simulation model using MT19937 and the analytical model in Equations 2 and show the differences in penetration rate and end node probabilities by **Table** 4. From this table, we have the same observations as for node probabilities. That is, the differences between $P_M(k;\bar{h})$ and $P(k;\bar{h})$ decrease to zero in the rate of $1/\sqrt{M}$, approximately.

From the comparison of the Monte Carlo simulation results with the analytical results, the convergence patterns observed for all three norms of errors agree with that predicted by the fundamental theorem of Monte Carlo simulations; we can conclude that the Monte Carlo simulation model is valid and converges to the analytical model in *(16)*. In this sense, these two models cross-validate each other.

5. CONCLUSION

In this paper, we discussed the conceptual framework for IIVC modeling and proposed a Monte Carlo simulation model for analyzing multihop connectivity of instantaneous inter-vehicle communication in a traffic stream. Although this study is not intended to compare RNGs, the MT19937 RNG proves to be the most desirable among the three RNGs for the Monte Carlo simulation model of IIVC since it is fast with attractive properties. From this study, we can see that the impact of vehicle movement on multihop connectivity can be reasonably ignored, since the model yields multihop connectivity results consistent with those in *(12)*, where vehicle mobility was considered. Based on the assumption of IIVC, the Monte Carlo simulation model is much more computationally efficient than models incorporating traffic simulators.

This study can be considered as a validation of the analytical models developed in *(16,17)*. Although simpler being without cell-membership specification, the Monte Carlo simulation model is as powerful and useful as the analytical models, since both provide node and end node probabilities. Moreover, with the simplification afforded by not considering hop probabilities, the Monte Carlo simulation model is more efficient for large-scale problems since the computational load increases linearly with the number of vehicles for the Monte Carlo simulation model, while quadratically for analytical models. Yet another obvious but important advantage of the new model is its simplicity in programming. Thus the new Monte Carlo simulation model can be applied independently for studying features of IIVC.

Given its efficiency, there are a number of extensions possible with the Monte Carlo simulation model to study IIVC. First, the Monte Carlo simulation model can be extended to incorporate location- or density-dependent transmission failure rates, wireless communication ranges, and penetration rates, by properly modifying the process of realizing Bernoulli trials and determining MFR communication chains. Second, it is easily adaptable to a network or roads — a problem that is probably intractable by analytical procedures. Third, the model could also be employed to develop a model of communication capacity of an IIVC system *(30)*, which is another important performance measurement. Finally, the model can be applied toward evaluating different protocols, such as in *(31,32)*, and especially their effectiveness under different traffic conditions and road network structures.

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TABLE 1. Comparison of node probabilities with RAND

M	1e3	1e4	1e5	1e6	1e7	1e8
$\left\| P_M(k;-1)-\mu \right\|_1$	1.8323e-4	5.8080e-5	1.8177e-5	5.6776e-6	1.8791e-6	7.0910e-7
$\left\| \cdot \right\|_2$	6.9097e-3	2.1902e-3	6.8543e-4	2.1410e-4	7.0861e-5	2.6740e-5
$\left\| \cdot \right\|_\infty$	2.2000e-2	8.1000e-3	2.3000e-3	7.4300e-4	2.3440e-4	6.6020e-5
$\left\| P_M(k;h)-P(k;h) \right\|_1$	2.2983e-5	8.4473e-6	2.2481e-6	7.0457e-7	3.0239e-7	7.9274e-8
$\left\| \cdot \right\|_2$	8.6669e-4	3.1854e-4	8.4773e-5	2.6569e-5	1.1403e-5	2.9894e-6
$\left\| \cdot \right\|_\infty$	1.2544e-2	6.0250e-3	1.2713e-3	4.9344e-4	1.6600e-4	5.2160e-5
CPU time (seconds)	1.4100e-1	5.6300e-1	4.8280	5.2438e1	4.8906e2	4.7941e3

TABLE 2. Comparison of node probabilities with MRG32k3a

M	1e3	1e4	1e5	1e6	1e7	1e8
$\|P_M(k;-1)-\mu\|_1$	1.8205e-4	5.7420e-5	1.8035e-5	5.8041e-6	1.8383e-6	5.7414e-7
$\|\cdot\|_2$	6.8650e-3	2.1653e-3	6.8007e-4	2.1887e-4	6.9320e-5	2.1650e-5
$\|\cdot\|_\infty$	2.1000e-2	7.6000e-3	2.4200e-3	7.8500e-4	2.5370e-4	7.4950e-5
$\|P_M(k;h)-P(k;h)\|_1$	2.3869e-5	7.6824e-6	2.5533e-6	7.5616e-7	2.7190e-7	7.8789e-8
$\|\cdot\|_2$	9.0007e-4	2.8970e-4	9.6283e-5	2.8514e-5	1.0253e-5	2.9711e-6
$\|\cdot\|_\infty$	1.8569e-2	5.6454e-3	1.3900e-3	4.6798e-4	1.9138e-4	4.4170e-5
CPU time (seconds)	2.3500e-1	1.6090	1.5891e1	1.5322e2	1.5328e3	1.5292e4

TABLE 3. Comparison of node probabilities with MT19937

M	1e3	1e4	1e5	1e6	1e7	1e8
$\left\Vert P_M(k;-1)-\mu\right\Vert_1$	1.8412e-4	5.7507e-5	1.8941e-5	5.7828e-6	1.8741e-6	5.8001e-7
$\left\Vert\cdot\right\Vert_2$	6.9431e-3	2.1686e-3	7.1425e-4	2.1806e-4	7.0671e-5	2.1872e-5
$\left\Vert\cdot\right\Vert_\infty$	2.7000e-2	8.0000e-3	2.3400e-3	1.0190e-3	2.5460e-4	7.8670e-5
$\left\Vert P_M(k;h)-P(k;h)\right\Vert_1$	2.3640e-5	8.0027e-6	2.5177e-6	8.1972e-7	2.2953e-7	7.4876e-8
$\left\Vert\cdot\right\Vert_2$	8.9145e-4	3.0178e-4	9.4939e-5	3.0911e-5	8.6553e-6	2.8235e-6
$\left\Vert\cdot\right\Vert_\infty$	1.5318e-2	4.5631e-3	1.6600e-3	4.6425e-4	1.2352e-4	5.0290e-5
CPU time (seconds)	1.5700e-1	9.5300e-1	7.7970	7.7594e1	6.9038e2	6.7585e3

TABLE 4. Comparison of end node probabilities with MT19937

M	1e3	1e4	1e5	1e6	1e7	1e8
$\lVert P_M(k;-1)-\mu \rVert_1$	1.1731e-4	3.8097e-5	1.2182e-5	3.5252e-6	1.1161e-6	3.5566e-7
$\lVert \cdot \rVert_2$	4.4236e-3	1.4366e-3	4.5938e-4	1.3293e-4	4.2086e-5	1.3412e-5
$\lVert \cdot \rVert_\infty$	2.6000e-2	1.0100e-2	3.0800e-3	9.7800e-4	2.6890e-4	7.6120e-5
$\lVert P_M(k;h)-P(k;h) \rVert_1$	1.1181e-5	3.1905e-6	1.0358e-6	3.0626e-7	1.2208e-7	3.0450e-8
$\lVert \cdot \rVert_2$	4.2163e-4	1.2031e-4	3.9060e-5	1.1549e-5	4.6037e-6	1.1483e-6
$\lVert \cdot \rVert_\infty$	8.2419e-3	2.1463e-3	6.8767e-4	2.4277e-4	1.0627e-4	1.9370e-5

FIGURE 1. A Monte Carlo simulation model of instantaneous inter-vehicle communication in a traffic stream

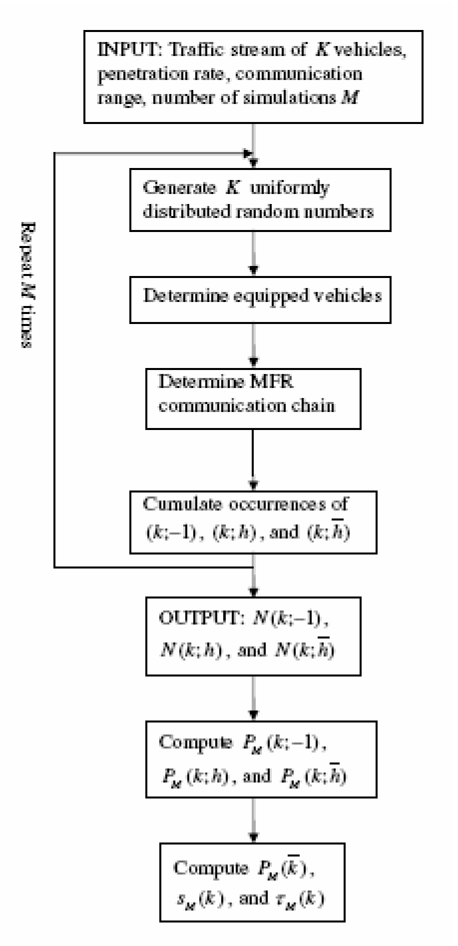

Figure 2. Comparison of connectivity for a uniform traffic stream: $\rho = 58$ veh/km, $\mu = 10\%$, and $M = 1000$; all four lines with marks are from Figure 1 of *(12)*, and the corresponding solid lines are from the Monte Carlo simulation model

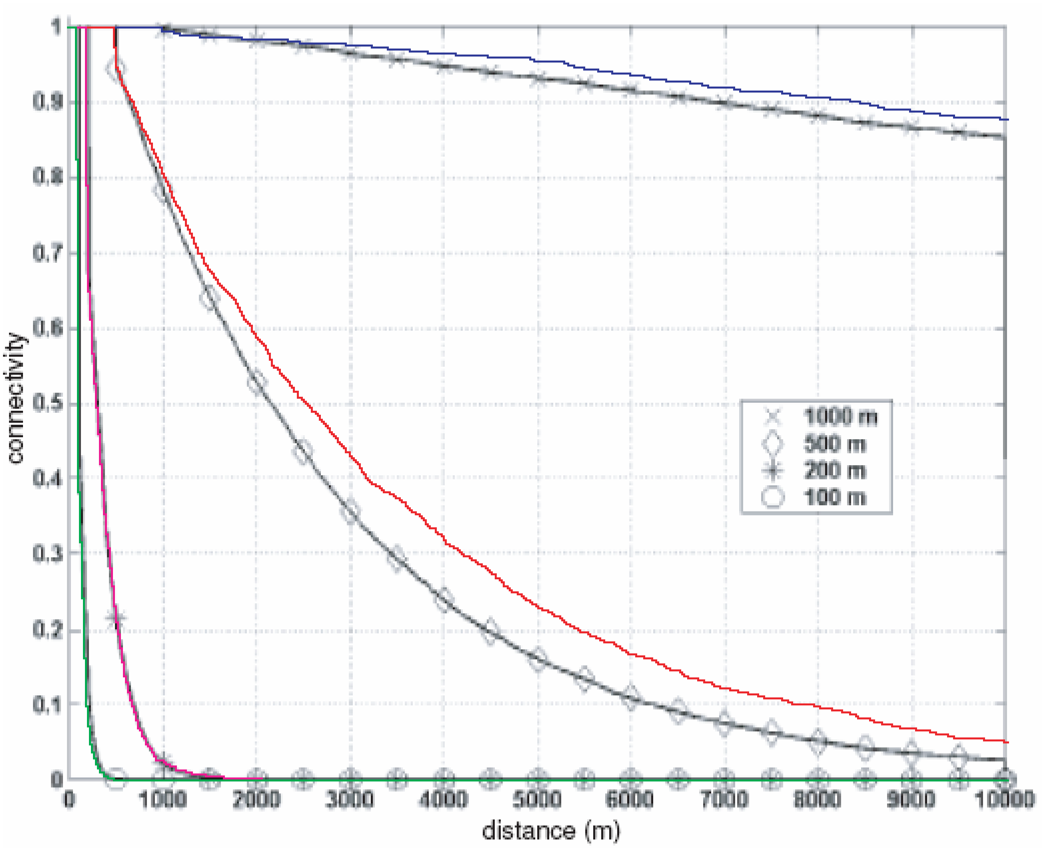

Figure 3. Comparison of connectivity for a uniform traffic stream: $\rho = 19$ veh/km, $\mu = 10\%$, and $M = 1000$; all four lines with marks are from Figure 2 of *(12)*, and the corresponding solid lines are from the Monte Carlo simulation model